Model Reduction Analysis of Activated Sludge Models using the concept of Extent of Reaction, Mass Transfer and Flow

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Highlights

- The concept of extent of reaction, flow and invariants are used to obtain reduced order model without any approximations.
- Reduced order model shows the exact dynamical behavior as the original ASM thereby reducing the computational complexity.
- Reduced order model is essential for control and optimization of large WWTPs.

1. Introduction

With the advent of industrialization, wastewater poses a threat to the environment and humankind. Sustainable development can be aided by wastewater treatment (WWT) by optimizing the quantities of nutrients like nitrogen, phosphorous and carbon while removing undesired nutrients. The International Water Association (IWA) created the Activated Sludge Model (ASM) series representing the biological treatment of wastewater from municipal and industrial streams. The main objective of this work is to develop a reduced order model of the system using the concepts of extents of reaction, flow and invariants. This approach can be used to reduce the number of differential equations in the original model of the system such that the reduced-order model maintains the identical dynamic behavior as that of the original models thereby reducing the computational complexity.

2. Methodology

For a reaction system involving *S* species, *R* reactions, *p* inlets and one outlet of the form

$$\dot{n}(t) = Q\tilde{S}_{in} - \frac{Q}{V}\, n(t) + N^T\rho(t)V, \;\; n(0) = n_0 \tag{1}$$

with $rank([N^T\ S_{in}\ n_0]) = R + p + 1$ can be linearly transformed as [1]

$$n \to [x_r\ x_{in}\ \lambda\ x_{iv}]^T = [S_0^T\ M_0^T\ q_0^T\ Q_0^T]n \tag{2}$$

with

$$S_0^T = S^T(I_S - n_0 q_0^T),\ M_0^T = M^T(I_S - n_0 q_0^T), q_0^T = \frac{1_{S-R-p}^T Q^T}{1_{S-R-p}^T Q^T n_0} \tag{3}$$

which converts *S*-dimensional mass vector into the *R*-dimensional extent of reactions (x_r), the *p*-dimensional extent of inlet flow (x_{in}), discounting variable (λ) to eliminate the effect of initial conditions, and the *(S-R-p-1)*-dimensional true invariants (x_{iv}).

$$\dot{x}_{r,i} = \rho_i V - \frac{Q}{V}x_{r,i}\ ,\ \ x_{r,i}(0) = 0,\ \ \forall i = 1, \dots, R \tag{4}$$

$$\dot{x}_{in,j} = Q - \frac{Q}{V}x_{in,j}\ ,\ \ x_{in,j}(0) = 0,\ \ \forall j = 1, \dots, p \tag{5}$$

$$\dot{\lambda} = -\frac{Q}{V}\lambda,\ \ \lambda(0) = 0 \tag{6}$$

$$\dot{x}_{iv,k} = 0\ ,\ \ x_{iv}(0) = 0,\ \forall k = 1 \dots q \tag{7}$$

The extent of outlet flow is calculated using discounting variable as $x_{out} = 1 - \lambda$. The invariant states (x_{iv}) provide the q ($q = S - (R + p + 1)$) invariant relationships which are independent of time. The mass of species can be reconstructed from the states of extents and discounting variable as follows

$$n(t) = N^T x_r(t) + \tilde{S}_{in}x_{in}(t) + n_0\lambda(t) \tag{8}$$

3. Results and discussion

The result shows the reduction of mass balance for batch and convective mass flows using the concept of extents of reaction, flow and the true invariants in a systematic manner (equation 4-8). The number of ODEs in the original model is reduced in the extent domain by removing the true invariant states (Table 1). The decrease in number of ODEs of ASM is more useful when applied to large systems such as BSM (five tanks in series) which helps in reducing the computational complexity of the model while ensuring that the dynamical behavior is the same.

Table 1: Model Reduction Analysis of ASM series[2]

	ASM1	**ASM2**	**ASM2d**	**ASM3**
Purpose	Used for Nitrogen removal Includes O_2 consumption, sludge production, nitrification and denitrification	Used for Nitrogen and Phosphorous removal Includes cell internal structure	Extension of ASM2 which includes Phosphorus Accumulating Organisms (PAOs) for denitrification	Includes endogenous respiration
State Variables	S_I, S_S, X_I, X_S, $X_{B,H}$, $X_{B,A}$, X_P, S_O, S_{NO}, S_{NH}, S_{ND}, X_{ND}, S_{ALK}	S_{O2}, S_F, S_A, S_{NH4}, S_{NO3}, S_{PO4}, S_I, S_{ALK}, S_{N2}, X_I, X_S, X_H, X_{PAO}, X_{PP}, X_{PHA}, X_{AUT}, X_{TSS}, X_{MeOH}, X_{MeP}	S_{O2}, S_F, S_A, S_{NH4}, S_{NO3}, S_{PO4}, S_I, S_{ALK}, S_{N2}, X_I, X_S, X_H, X_{PAO}, X_{PP}, X_{PHA}, X_{AUT}, X_{TSS}, X_{MeOH}, X_{MeP}	S_{O2}, S_I, S_S, S_{NH4}, S_{N2}, S_{NOX}, S_{ALK}, X_I, X_S, X_H, X_{STO}, X_A, X_{SS}
No. of. Processes	8	19	21	12
No. of. ODEs	13	19	19	13
Stoichiometric Matrix	$N \in R^{8\times13}$	$N \in R^{19\times19}$	$N \in R^{21\times19}$	$N \in R^{12\times13}$
Reduced Order Model (using extent of reaction, mass transfer and flow)				
Batch System				
No. of. ODEs	8	14	14	9
Continuous System				
No. of. ODEs	10	16	16	11

4. Conclusions

The model reduction analysis was carried out for ASM series which includes ASM1, ASM2, ASM2d and ASM3 using the concept of extent of reaction, flow and true invariants. The reduced order model shows it is useful when applied to large WWTPs involving several units. The future work is to simulate the dynamical behavior of the original model and reduced order model for ASM series.

References

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Keywords

ASM, Wastewater Treatment, Model Reduction, Extents of Reaction and Flow.